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# Comment on "Quasiparticle Anisotropy and Pseudogap Formation from the Weak-Coupling Renormalization Group Point of View"

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In their recent Letter, Katanin and Kampf [1] (KK) reported numerical results for the self-energy $\Sigma(\boldsymbol{k};\varepsilon)$, $\varepsilon \in \mathbb{R}$, of the single-band Hubbard Hamiltonian (hereafter 'single-band' will be implicit) in two space dimensions (i.e. $d = 2$), obtained through employing the functional renormalization-group (fRG) formalism (see references in [1], and [2]) at the one-loop level. Several of the results by KK are in full conformity with the *exact* formal results reported earlier in [3, 4, 5]. This, as we shall elaborate below, strengthens one's confidence in the reliability of the fRG in dealing with models of strongly-correlated fermions.

*(i) The quasi-particle (qp) weight $Z_{\mathrm{F}}$ at the Fermi surface.* — Amongst other things, KK showed that [1] "At van Hove (vH) band fillings and at low temperatures, the quasiparticle weight along the Fermi surface (FS) continuously vanishes on approaching the $(\pi, 0)$ point ...." In [3] we have obtained a general expression for the momentum-distribution function $\mathsf{n}(\boldsymbol{k})$ at $\boldsymbol{k} = \boldsymbol{k}_{\mathrm{F}}^{\mp}$, i.e. infinitesimally in- and outside Fermi sea (here we suppress the spin indices encountered in [3, 4, 5]), which on the basis of the available numerical results at the time we have shown to reduce to (Eq. (99) in [3]): $\mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{\mp}) = (a+Ub^{\mp})/(a+2Ub^{\mp})$, where $a$ and $b^{\mp}$ are components of vectors $\boldsymbol{a}(\boldsymbol{k}_{\mathrm{F}}) \equiv \boldsymbol{\nabla}_{\boldsymbol{k}}\varepsilon_{\boldsymbol{k}}|_{\boldsymbol{k}=\boldsymbol{k}_{\mathrm{F}}}$ and $\boldsymbol{b}(\boldsymbol{k}_{\mathrm{F}}^{\mp})$ along the outward unit vector normal to Fermi surface $\mathcal{S}_{\mathrm{F}}$ at $\boldsymbol{k} = \boldsymbol{k}_{\mathrm{F}}$; here $\varepsilon_{\boldsymbol{k}}$ is the non-interacting energy dispersion and $\boldsymbol{b}(\boldsymbol{k})$ the gradient of a well-defined scalar ground-state (GS) correlation function. Stability of the latter GS is tantamount to the satisfaction of [3] $b^{-} > a/(U\Lambda^{-})$, $b^{+} < -a/U$, where $\Lambda^{-} \equiv \mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{-})/[1 - \mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{-})] > 0$. Our above expression for $\mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{\mp})$ makes explicit that (a) $\mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{\mp}) \gtreqless 1/2$, (b) $\mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{\mp}) \to 1/2$, i.e. $Z_{\mathrm{F}} \equiv \mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{-}) - \mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{+}) \to 0$, for $Ub^{\mp} \to \infty$, and (c) $\mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{\mp}) = 1/2$, i.e. $Z_{\mathrm{F}} = 0$, for $a = 0$, that is for $\boldsymbol{k}_{\mathrm{F}}$ at vH points. In Fig. 1 we compare our results for $Z_{\mathrm{F}}$ with those determined by KK [1], *disregarding* the dependence of $b^{\mp}$ on the direction of $\boldsymbol{k}_{\mathrm{F}}$.

*(ii) The single-particle spectral function $A(\boldsymbol{k};\varepsilon)$ for $\boldsymbol{k}$ in the pseudogap (PG) region.* — KK observed that [1] "The qp weight suppression [for $\boldsymbol{k}_{\mathrm{F}} \to (\pi, 0)$] is accompanied by the growth of two additional incoherent peaks in the spectral function, from which an anisotropic pseudogap originates." On general theoretical grounds, in [3, 4] we have shown that the *singular* nature of $\mathsf{n}(\boldsymbol{k})$ at *all* $\boldsymbol{k} \in \mathcal{S}_{\mathrm{F}}^{(0)}$ (see *(iii)* below) implies that (see Sec. 10 in [3])

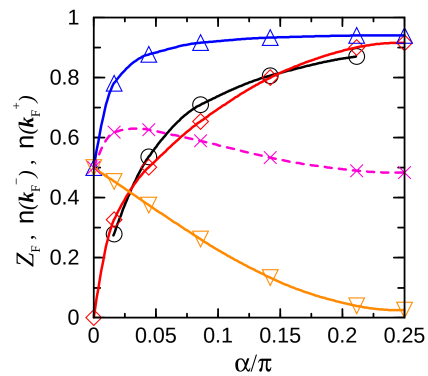


FIG. 1: $Z_{\mathrm{F}}$ as calculated by KK (◯) and according to the exact expression in the text with $b^{-} = 0.0912$, $b^{+} = -1.4158$ (◊), corresponding to $U/t = 2$ and the vH filling associated with $t'/t = 0.1$. $\alpha \equiv \widehat{\boldsymbol{k}_{\mathrm{F}}, (\pi, 0)}$. The apparent deviation between the two results reflects the isotropy of $b^{\mp}$ *assumed* here. Using the above $b^{\mp}$, we also present $\mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{-})$ (△) and $\mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{+})$ (▽) obtained from the exact expressions in the text, as well as $[\mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{-}) + \mathsf{n}(\boldsymbol{k}_{\mathrm{F}}^{+})]/2$ (×) whose deviation from 1/2 in the present case is indicative of the underlying metallic state *not* being a Fermi liquid [3] (leaving aside $Z_{\mathrm{F}} = 0$ at $\alpha = 0$).

$Z_{\mathrm{F}} \to 0$, for $\boldsymbol{k} \to$ PG, *must* necessarily be accompanied by at least two *resonant* peaks (to be distinguished from qp peaks) in $A(\boldsymbol{k};\varepsilon)$, one *strictly below* and one *strictly above* the Fermi energy $\varepsilon_{\mathrm{F}}$. Here $\mathcal{S}_{\mathrm{F}}^{(0)}$ is the Fermi surface associated with $\varepsilon_{\boldsymbol{k}}$ (see *(iii)* below).

*(iii) Fermi surface non-deformation.* — For models involving solely contact-type interaction, we have shown that [3] $\mathcal{S}_{\mathrm{F}} \subseteq \mathcal{S}_{\mathrm{F}}^{(0)}$; PG consists of those points of $\mathcal{S}_{\mathrm{F}}^{(0)}$, if any, which do not belong to $\mathcal{S}_{\mathrm{F}}$ [3, 4]. Interestingly, $\mathcal{S}_{\mathrm{F}} \subseteq \mathcal{S}_{\mathrm{F}}^{(0)}$ turns out to be the working hypothesis for many calculations, amongst which those by KK [1].

We should like to emphasize that the above-mentioned results, cited from [3, 4, 5], are *not* restricted to the weak-coupling limit; the only constraint for the validity of these results is the uniformity of the underlying GSs. We further point out that, *in principle*, depending on the values of $d$, $U/t$, $t'/t$, etc., some other 'universality classes' for the uniform metallic GSs of the Hubbard Hamiltonian (explicitly considered in [3, 4]) may/do become viable than that dealt with in this Comment.

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[2] P. Kopietz, and T. Busche, Phys. Rev. B **64**, 155101 (2001).
[3] B. Farid, Phil. Mag. **83**, 2829 (2003). [cond-mat/0211244]
[4] B. Farid, Phil. Mag. **84**, 109 (2004). [cond-mat/0304350]
[5] For cases where $Z_{\mathrm{F}} \neq 0$, some of the pertinent results are more conveniently deduced by the approach in B. Farid, Phil. Mag. **84**, 909 (2004). [cond-mat/0308090]